STA 130A – Homework 0

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Instructions: This assignment is for your self-assessment and practice only. It will *not* be collected or graded, nor will solutions be provided. It reviews calculus and introductory-statistics background that will be used in STA 130A. Problems 1–3 review sums and series, integration, differentiation, and multivariable calculus. Problem 4 is an optional preview[1] of double integrals and changes of variables, which will become useful when we study joint continuous distributions. Problems 5–6 review samples, summary statistics, sampling variability, and confidence intervals in preparation for the estimation material near the end of the course.

The goal of this self-assessment is not to get every part correct on the first attempt, but to identify topics worth reviewing before the course begins. If several parts of Problems 1–3 or Problems 5–6 feel difficult, review the relevant material from your prerequisite calculus and statistics courses before the quarter starts, and bring questions to discussion section or office hours during the first week.

Problem 1. Sums and Series

(a) For a real number $r \neq 1$, evaluate the finite geometric sum (as a function of r and n)

$$\sum_{k=0}^{n} r^k.$$

Then determine its limit as $n \to \infty$ when $|r| < 1$.

(b) Using the identity

$$\sum_{k=0}^{\infty} r^k = \frac{1}{1-r}, \qquad |r| < 1,$$

find closed forms for

$$\sum_{k=1}^{\infty} k r^{k-1} \qquad \text{and} \qquad \sum_{k=1}^{\infty} k r^k.$$

Hint: Differentiate the geometric-series identity with respect to r.

(c) For each expression below, state whether it is a finite sum or an infinite series. For each finite sum, compute its value. For each infinite series, determine whether it converges, and if it does, compute its value.

$$\sum_{k=1}^{100} k, \qquad \sum_{k=1}^{100} 2^{-k}, \qquad \sum_{k=1}^{\infty} k^2, \qquad \sum_{k=1}^{\infty} 3^{-k}, \qquad \sum_{k=1}^{\infty} \frac{1}{k}, \qquad \sum_{k=1}^{\infty} (-1)^k.$$

[1] Double integrals and Jacobians are not assumed prerequisite material for STA 130A. You may skip Problem 4 for now and return to it later, and difficulty with that problem alone does not indicate that you are inadequately prepared for the course.

Problem 2. One-variable Calculus

Let

$$f(x) = \begin{cases} e^{-x}, & x \geq 0, \\ 0, & x < 0. \end{cases}$$

(a) Verify that

$$\int_{-\infty}^{\infty} f(x)\,dx = 1.$$

(b) For $t \in \mathbb{R}$, compute the function

$$F(t) = \int_{-\infty}^{t} f(x)\,dx.$$

Hint: Give your answer as a piecewise formula.

(c) Compute

$$\int_{0}^{\infty} xe^{-x}\,dx.$$

(d) For which real numbers s does

$$\int_{-\infty}^{\infty} e^{sx} f(x)\,dx$$

converge? Compute its value when it converges.

Problem 3. Multivariable Calculus

Let

$$g(x, y) = x^2 + xy + e^{x-2y}.$$

(a) Compute $\frac{\partial}{\partial x} g(x, y)$, $\frac{\partial}{\partial y} g(x, y)$, and $\frac{\partial^2}{\partial x \partial y} g(x, y)$.

(b) Compute $\nabla g(0, 0)$.

(c) Define

$$h(u, v) = g(u + v, u - v).$$

Compute $\frac{\partial}{\partial u} h(u, v)$ and $\frac{\partial}{\partial v} h(u, v)$.

(d) Define

$$c(t) = g(t, 1 - t).$$

Compute $c'(t) \coloneqq \frac{d}{dt} c(t)$ in two ways:

(i) first by substituting $x = t$ and $y = 1 - t$ into g and differentiating directly;

(ii) then by using the chain rule.

Verify that your two answers agree.

Problem 4. Optional Preview: Double Integrals and Change of Variables

This problem previews tools that will be useful for joint continuous distributions. These tools are not assumed prerequisite knowledge and will be reviewed as needed during the course.

(a) Sketch the region

$$R = \{(x, y) : 0 < y < x < 1\}.$$

Write

$$\iint_R (x + 2y)\, dA$$

as an iterated integral in both orders, and evaluate it in one of the two orders.

(b) Compute

$$\iint_{x^2+y^2\le 1} (x^2 + y^2)\, dA$$

using polar coordinates.

(c) Consider the change of variables $T : (x, y) \mapsto (u, v)$ given by

$$u = x + y, \qquad v = x - y.$$

(i) Solve for x and y in terms of u and v.

(ii) Describe the image of the unit square under T, namely, $U = T([0, 1] \times [0, 1])$, in the (u, v)-plane.

(iii) Compute the Jacobian determinant

$$\frac{\partial(x, y)}{\partial(u, v)}.$$

(iv) Rewrite and evaluate the following as an integral in (u, v)-coordinates:

$$\iint_D 1\, dA, \qquad \text{where} \qquad D = \{(x, y) : x > 0,\ y > 0,\ x + y < 1\}.$$

Problem 5. Samples, Parameters, and Statistics

Suppose that the following observations constitute a sample:

$$2, \qquad 4, \qquad 4, \qquad 5, \qquad 10.$$

(a) Compute the sample mean

$$\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

and the sample variance

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x})^2.$$

(b) Briefly explain the distinction between a population parameter and a sample statistic. If the observations are sampled from a population with mean μ, which of μ and $\overline{x}$ is a parameter, and which is a statistic?

(c) Let $X_1, \ldots, X_n$ be independent and identically distributed random variables satisfying

$$\mathbb{E}[X_i] = \mu, \qquad \mathrm{Var}(X_i) = \sigma^2,$$

and define

$$\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i.$$

Using the usual rules for expectation and variance, verify that

$$\mathbb{E}[\overline{X}] = \mu, \qquad \mathrm{Var}(\overline{X}) = \frac{\sigma^2}{n}.$$

(d) How does the standard deviation of $\overline{X}$ change if the sample size is multiplied by four?

Problem 6. Estimation and Confidence Intervals

Suppose that each individual in a population either has or does not have a certain characteristic. Let p denote the unknown population proportion having that characteristic. In an independent random sample of $n = 100$ individuals, 40 have the characteristic.

(a) Identify the population parameter, a natural estimator of that parameter, and the observed value of the estimator in this sample.

(b) Let

$$\widehat{p} = \frac{1}{n}\sum_{i=1}^{n} X_i,$$

where $X_i = 1$ if individual i has the characteristic and $X_i = 0$ otherwise. Compute the estimated standard error

$$\widehat{\mathrm{SE}}(\widehat{p}) = \sqrt{\frac{\widehat{p}(1-\widehat{p})}{n}}.$$

(c) Verify that

$$n\widehat{p} \geq 10 \qquad \text{and} \qquad n(1-\widehat{p}) \geq 10.$$

Then compute the approximate 95% confidence interval

$$\widehat{p} \ \pm\ 1.96\,\widehat{\mathrm{SE}}(\widehat{p}).$$

(d) Which of the following is the standard frequentist interpretation of the confidence level? Briefly explain your answer.

(i) After this interval has been computed, there is a 95% probability that the fixed parameter p lies in this particular interval.

(ii) If the sampling procedure were repeated many times and a confidence interval were constructed in the same way from each sample, approximately 95% of those intervals would contain p.

What to Do If You Struggle

- **Review:** Review the relevant material from your prerequisite calculus and introductory-statistics courses.
- textbfAttend the first discussion section: Bring questions about these prerequisite concepts to the first discussion section (Tue, September 29, 2026) so that the TA can help review prerequisite concepts and answer questions.
- **Ask for help:** If multiple areas are unclear, contact the instructor or TA, or form a study group with your classmates.