STA 130A – Homework 1

Submission due: Thu, October 1 at 11:59 PM PT

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Instructions: Upload a single PDF file to Canvas/Gradescope ("Homework 1" under "Assignments"). Name the file using the prefix of your UC Davis email ID and the homework number (e.g., `dgsong_hw1.pdf`). Include "STA 130A," your name, and the last four digits of your student ID on the front page. No late submissions will be accepted; any submission received after the deadline will receive 0 points. For full submission requirements and the late submission policy, see the syllabus.

Problem 1 (30 points in total).

(a) **(10 points)** [BT08, Problem 1, p. 53]

(b) **(10 points)** Let $S_1, S_2, \ldots$ be subsets of a common sample space Ω. Prove De Morgan's laws:

$$\left(\bigcup_{n=1}^{\infty} S_n\right)^c = \bigcap_{n=1}^{\infty} S_n^c, \qquad \left(\bigcap_{n=1}^{\infty} S_n\right)^c = \bigcup_{n=1}^{\infty} S_n^c.$$

(c) **(10 points)** [BT08, Problem 2-(a) & (b), p. 53]

Problem 2 (30 points in total).

(a) **(10 points)** [BT08, Problem 5, p. 54]

(b) **(10 points)** [BT08, Problem 6, p. 54]

(c) **(10 points)**[Adapted from [BT08, Problem 7, p. 54]. A three-sided die with faces labeled $1, 2, 3$ is rolled repeatedly until the first time that an odd number is obtained, if one ever occurs. Describe a sample space that records the full sequence of rolls. You may use set notation or a clear verbal description.

Problem 3 (20 points in total).

(a) **(10 points)** [BT08, Problem 9, p. 54]

(b) **(10 points)** [BT08, Problem 10, p. 54–55]

Problem 4 (20 points in total + 10 bonus points).

(a) **(10 points)** [BT08, Problem 15, p. 56]

(b) **(10 points)** [BT08, Problem 16, p. 57]

(c*) (10 bonus points)

A coin is tossed twice, with the four ordered outcomes HH, HT, TH, and TT equally likely. A friend reports information about the tosses using one of the following rules:

- **Rule I:** Your friend looks at both tosses and reports "Heads" if at least one toss is heads, and "No heads" otherwise.
- **Rule II:** Your friend chooses one of the two tosses, each with probability $1/2$, independently of the toss results. Your friend reports "Heads" or "Tails" according to the chosen toss, without telling you which toss was chosen.

In each case, you know which rule is being used and hear the report "Heads." For each rule, find the conditional probability that both tosses were heads. Explain why the answers can differ even though the same truthful word was reported.

References

[BT08] Dimitri Bertsekas and John N Tsitsiklis. *Introduction to probability*, volume 1. Athena Scientific, 2nd edition, 2008.