

STA 131A – Homework 0, due: Never

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Instructions: This assignment is for your self-assessment and practice only. It will *not* be collected or graded, nor will solutions be provided. It reviews calculus and linear algebra background that will be used repeatedly in STA 131A, especially material from MAT 21B/C and MAT 22A, together with a brief check on some MAT 21D material. The symbol (♠) indicates topics drawn mainly from MAT 21D; do not worry too much if you find those parts difficult, but do take them as a sign that some review would be helpful before we reach joint continuous distributions later in the quarter. If you have not yet taken MAT 21D, you may skip Problem 4 for now and return to it later; it is included mainly to flag material that will be useful later in the quarter.

The goal of this self-assessment is not to get every part correct on the first attempt, but to identify topics worth reviewing before the course begins. If several parts feel difficult, review your MAT 21B/C, MAT 22A, and, if relevant, MAT 21D notes before the quarter starts, and bring questions to discussion section or office hours during the first week.

Problem 1. Sums and Series

- (a) For a real number $r \neq 1$, evaluate the finite geometric sum (as a function of r and n)

$$\sum_{k=0}^n r^k.$$

Then determine its limit as $n \rightarrow \infty$ when $|r| < 1$.

- (b) Using the identity

$$\sum_{k=0}^{\infty} r^k = \frac{1}{1-r}, \quad |r| < 1,$$

find closed forms for

$$\sum_{k=1}^{\infty} k r^{k-1} \quad \text{and} \quad \sum_{k=1}^{\infty} k r^k.$$

Hint: Differentiate the geometric-series identity with respect to r .

- (c) For each expression below, state whether it is a finite sum or an infinite series. For each finite sum, compute its value. For each infinite series, determine whether it converges, and if it does, compute its value.

$$\sum_{k=1}^{100} k, \quad \sum_{k=1}^{100} 2^{-k}, \quad \sum_{k=1}^{\infty} k^2, \quad \sum_{k=1}^{\infty} 3^{-k}, \quad \sum_{k=1}^{\infty} \frac{1}{k}, \quad \sum_{k=1}^{\infty} (-1)^k.$$

Problem 2. One-variable Calculus

Let

$$f(x) = \begin{cases} e^{-x}, & x \geq 0, \\ 0, & x < 0. \end{cases}$$

(a) Verify that

$$\int_{-\infty}^{\infty} f(x) dx = 1.$$

(b) For $t \in \mathbb{R}$, compute the function

$$F(t) = \int_{-\infty}^t f(x) dx.$$

Hint: Give your answer as a piecewise formula.

(c) Compute

$$\int_0^{\infty} x e^{-x} dx.$$

(d) For which real numbers s does

$$\int_{-\infty}^{\infty} e^{sx} f(x) dx$$

converge? Compute its value when it converges.

Problem 3. Multivariable Calculus

Let

$$g(x, y) = x^2 + xy + e^{x-2y}.$$

(a) Compute $\frac{\partial}{\partial x}g(x, y)$, $\frac{\partial}{\partial y}g(x, y)$, and $\frac{\partial^2}{\partial x \partial y}g(x, y)$.(b) Compute $\nabla g(0, 0)$.

(c) Define

$$h(u, v) = g(u + v, u - v).$$

Compute $\frac{\partial}{\partial u}h(u, v)$ and $\frac{\partial}{\partial v}h(u, v)$.

(d) Define

$$c(t) = g(t, 1 - t).$$

Compute $c'(t) := \frac{d}{dt}c(t)$ in two ways:

- (i) first by substituting $x = t$ and $y = 1 - t$ into g and differentiating directly;
- (ii) then by using the chain rule.

Verify that your two answers agree.

Problem 4. (♠) Double Integrals and Change of Variables

(a) Sketch the region

$$R = \{(x, y) : 0 < y < x < 1\}.$$

Write

$$\iint_R (x + 2y) dA$$

as an iterated integral in both orders, and evaluate it in one of the two orders.

(b) Compute

$$\iint_{x^2+y^2 \leq 1} (x^2 + y^2) dA$$

using polar coordinates.

(c) Consider the change of variables $T : (x, y) \mapsto (u, v)$ given by

$$u = x + y, \quad v = x - y.$$

(i) Solve for x and y in terms of u and v .(ii) Describe the image of the unit square under T , namely, $U = T([0, 1] \times [0, 1])$, in the (u, v) -plane.

(iii) Compute the Jacobian determinant

$$\frac{\partial(x, y)}{\partial(u, v)}.$$

(iv) Rewrite and evaluate the following as an integral in (u, v) -coordinates:

$$\iint_D 1 dA, \quad \text{where} \quad D = \{(x, y) : x > 0, y > 0, x + y < 1\}.$$

Problem 5. Linear Algebra

Let

$$u = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}, \quad v = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}, \quad A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}, \quad b = \begin{pmatrix} 5 \\ 3 \end{pmatrix},$$

and

$$a = \begin{pmatrix} 1 \\ -2 \end{pmatrix}, \quad \Sigma = \begin{pmatrix} 4 & -1 \\ -1 & 2 \end{pmatrix}.$$

(a) Compute $u \cdot v$ and $\|u\|^2$.(b) Solve the system $Ax = b$ by row reduction.(c) Compute Σa and then $a^\top \Sigma a$.

(d) Let

$$\mathcal{B} = (b_1, b_2) \quad \text{where} \quad b_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad b_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix},$$

be an ordered basis of \mathbb{R}^2 .(i) Find the coordinates of $x = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ with respect to the basis \mathcal{B} .(ii) Write down the change-of-coordinate matrix from \mathcal{B} -coordinates to standard coordinates, and vice versa. Use the latter to verify your coordinate vector.

What to Do If You Struggle

- **Review:** Review the relevant MAT 21B/21C/22A material, and MAT 21D material if needed.
- **Attend discussion sessions:** In the first week (Thu, April 2, 2026), the TA will help review prerequisite concepts and answer questions.
- **Ask for help:** If multiple areas are unclear, contact the instructor or TA, or form a study group with a few classmates.