STA 131A – Homework 2

Submission due: Tue, April 14 at 11:59 PM PT

Instructor: Dogyoon Song

Instructions: Upload a single PDF file to Gradescope via Canvas ("Homework 2" under "Assignments"). Name the file using the prefix of your UC Davis email ID and the homework number (e.g., `dgsong_hw2.pdf`). Include "STA 131A," your name, and the last four digits of your student ID on the front page. No late submissions will be accepted; any submission received after the deadline will receive 0 points. For full information about submission requirements and the late submission policy, see the syllabus.

Problem 1 (30 points in total).

(a) (12 points) [BT08, Chapter 1, Problem 14, p. 56]

(b) (6 points) Let A, B, C be events with $P(B), P(C) > 0$. Assume $B \subseteq C$ and $A \cap B = A \cap C$. Prove

$$P(A \mid B) \geq P(A \mid C).$$

(c) (6 points) Let A and B be events with $0 < P(A) < 1$ and $0 < P(B) < 1$. Prove that

$$P(A \mid B) > P(A) \qquad \text{if and only if} \qquad P(B \mid A) > P(B \mid A^c).$$

(d) (6 points) Let A and B be independent events. Prove that if $A \subseteq B$, then either $P(A) = 0$ or $P(B) = 1$.

Problem 2 (25 points in total).

(a) (7 points) [BT08, Chapter 1, Problem 30, p. 60]

(b) (10 points) [BT08, Chapter 1, Problem 36, pp. 61–62]

(c) (8 points) [BT08, Chapter 1, Problem 39, p. 62]

Problem 3 (25 points in total).

(a) (7 points) [BT08, Chapter 1, Problem 49, p. 67]

(b) (8 points) [BT08, Chapter 1, Problem 50, pp. 67–68]

(c) (10 points) [BT08, Chapter 1, Problem 51, p. 68]

Problem 4 (20 points in total).

(a) (10 points) [BT08, Chapter 2, Problem 3, p. 119]

(b) (10 points) [BT08, Chapter 2, Problem 7, p. 120]

Problem 5* (up to 10 bonus points).

Each subproblem is worth 5 bonus points. At most two bonus subproblems will count, for a maximum of 10 bonus points.

(a) (5 bonus points) [BT08, Chapter 1, Problem 24, p. 58]

(b) (5 bonus points) [BT08, Chapter 1, Problem 40, p. 62]

(c) (5 bonus points) [BT08, Chapter 1, Problem 60, p. 69]

(d) (5 bonus points) [BT08, Chapter 2, Problem 9, p. 120]

References

[BT08] Dimitri Bertsekas and John N Tsitsiklis. *Introduction to probability*, volume 1. Athena Scientific, 2nd edition, 2008.