STA 131A: Introduction to Probability Theory

Lecture 1: Introduction and Overview

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Agenda

- Course overview
- Course logistics
- A first look at probability

A motivating dialogue

A: Hey, what is the probability of rain tomorrow?

B: I hope it doesn't rain, but we will know tomorrow.

A: Yes, but what is the probability that it will rain?

B: Each day is different, so we have to wait and see.

A: Out of 100 similar days, on about how many would you expect rain?

B: I told you, sometimes it rains, sometimes it doesn't.

A: If you had to bet whether it will rain or not, which side would you bet?

B: I'd bet it won't rain.

A: Ok, then would you be willing to lose $5 if it rains and win $1 if it doesn't?

B: ... probably not. Stop it.

What is probability?

Probability is a mathematical language for reasoning under uncertainty

Two common ways to interpret probability are:

- Long-run relative frequency of occurrences
  - Percentage of similar days on which it rains
  - Percentage of similar patients for whom a treatment works
  - Percentage of times an asset price increases under similar conditions
  - ...
- Subjective belief
  - Choices under uncertainty can reveal a person's subjective probabilities

Either way, probability gives us a coherent language for reasoning under uncertainty, useful in various fields including statistics, science, engineering, medicine, finance, etc.

STA 131A and objectives

STA 131 sequence introduces probability and mathematical statistics

- **STA 131A**: Introduction to **probability theory**
- **STA 131B/C**: Introduction to mathematical statistics (sampling, estimation, inference, testing, linear models, ...)

Our focus is on learning probability theory from **first principles**:

- Understand foundational probabilistic concepts and notation
- Learn common probabilistic models and core analytical tools

This course develops the mathematical framework for coherent probability calculations

- We will not dwell on philosophical debates or measure-theoretic foundations

Course logistics

- **Instructor:** Dogyoon Song
  - Email: dgsong@ucdavis.edu
  - Office hours: Wed 2:30–3:30 PM (or by appointment) at MSB 4220
- **TA:** Wonjun Seo
  - Email: wseo@ucdavis.edu
  - Office hours: TBA
- **Lectures:** Mon/Wed/Fri 10:00–10:50 AM at Wellman Hall 226
- **Discussions:** Thu 3:10-4:00 PM (Hart 1130) / 4:10-5:00 PM (Wellman 115)
- **Online platforms**
  - Course webpage: Lecture notes, homework, and supplementary materials
  - Canvas: Homework submission, solutions, grades, and discussion materials
  - Piazza: Announcements and discussion
  - Email: Private matters only (please **do not** message on Canvas)

Course content & prerequisites

Course content:

- Probability basics
- Discrete and continuous random variables
  - Probability mass/density functions
  - Expectation, variance, moments
  - Joint and conditional distributions
- Further topics on random variables
  - Derived distributions
  - Covariance and correlation
  - Moment generating functions
- Limit theorems
  - Markov and Chebyshev inequalities
  - The (weak) law of large numbers
  - The central limit theorem

Prerequisites:

- MAT 021C (C- or better)
- MAT 022A or MAT 027A or MAT 067 (C- or better)
- MAT 021D strongly recommended

These prerequisites are enforced

- If you don't meet prerequisites, please submit a petition ASAP
- Try "Homework 0" for a quick self-assessment

“Homework 0” for self-assessment

- Complete the “Homework 0” ASAP for your self-assessment if you haven’t already
- It reviews key material from calculus and linear algebra (especially MAT 21C and MAT 22A), plus some MAT 21D topics
- It will not be collected or graded, and no solutions will be provided
- If you struggle with any part, review your notes/resources, **attend discussion sections this week** (Thu, April 2, 2026) or bring questions to office hours this week

Grading

- **Homework**: 40%
  - 7 homework assignments, excluding "Homework 0"
  - Assigned on Wednesday morning, due next Tuesday 11:59 pm PT
  - One homework with the lowest score can be dropped
  - **No late homework accepted for any reason**
- **Midterm exams**: 25%
  - Two in-class midterms (Fri, April 24 & Fri, May 15)
  - The lower can be dropped
  - **No make-up exams offered**
- **Final exam**: 35%
  - Thursday, June 11, 1:00-3:00 PM
- **Participation**: up to 3% extra through in-person participation in lecture/discussion

See syllabus for full details and additional information (textbook, course policies, etc.)

Probability in everyday examples

- Coin toss
  - Possible outcomes are Head or Tail; each has probability 0.5
- Die roll
  - Possible outcomes $\{1, 2, \ldots, 6\}$; each has probability $1/6$
- Y chromosomes in the US childbirths[1]
  - About 51.2% of births are to babies with Y chromosomes, and 48.8% do not
  - A rough empirical probability of having a baby with a Y chromosome is 0.512
- Commute time
  - Commute may take 30 minutes on average, but can vary with traffic, weather, etc.
- Subjective probabilistic beliefs
  - You may personally estimate the likelihood of a stock price rising or falling, based on your own analysis or expert opinions
  - A scholar may believe an artifact is authentic with probability 90% and fake with 10%

[1] Source: CDC National Vital Statistics Reports, Births: Final Data for 2023

Formalizing probability (Teaser for next lecture)

- **Outcome:** A possible result of an experiment or trial
- **Sample space**: the set of all possible outcomes, often denoted by Ω
  - e.g., $\{H, T\}$, $\{1, 2, 3, 4, 5, 6\}$
- **Event:** a subset of Ω
  - e.g., for $\Omega = \{H, T\}$: $\emptyset$, $\{H\}$, $\{T\}$, $\{H, T\}$
  - e.g., for $\Omega = \{1, 2, 3, 4, 5, 6\}$: $\{6\}$, $\{1, 2\}$, $\{2, 4, 6\}$
- **Probability[2]:** a map P that assigns a number in $[0, 1]$ to each event such that
  - $P(\Omega) = 1$;
  - For pairwise disjoint events $A_1, A_2, \dots$, $P\left(\bigcup_i A_i\right) = \sum_i P(A_i)$

Probability theory makes extensive use of set notation and set operations!

[2] A fully rigorous measure-theoretic formulation is beyond the scope of STA 131A

Where we are headed

- **Throughout the course**:
  - Understand foundational probabilistic concepts and notation
  - Learn common probabilistic models and core analytical tools
  - Practice rigorous mathematical thinking in probability theory
- **Immediate next steps (Weeks 1–2)**:
  - Set theory and probabilistic models
  - Basic probability laws
  - Counting
- **Before next lecture**:
  - Complete “Homework 0” and seek help if needed